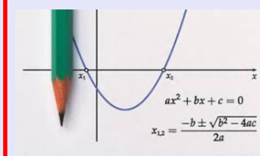
Math 125

Spring 2022

Lecture 14



Class QZ 11

Solve by addition method:

$$\begin{cases} 2x - 3y = -1 & (2) \\ 3x + 2y = 5 & (3) \end{cases} \Rightarrow \begin{cases} 4x \cancel{-6y} = -2 \\ 9x \cancel{+6y} = 15 \end{cases}$$

$$13x = 13 \qquad \boxed{x=1}$$

$$\underbrace{2(1)}_{2} - 3y = -1 \qquad -3y = -3 \qquad \boxed{y=1}$$

$$(1,1), \ \{(1,1)\}$$

I deposited \$5000 in 3 accounts for one Year, Simple interest.
One pays 2%, one pays 4%, and last one pays 5% interest. Total interest made was \$214
money invest in 5% rate was \$600 less than three times the money invested at 4% account.
How much per account?

$x \rightarrow 2\%$ rate
$y \rightarrow 4\%$ rate
$z \rightarrow 5\%$ rate

$$100\begin{cases} x + y + z = 5000 \\ 2\% x + 4\% y + 5\% z = 214 \\ z = 3y - 600 \end{cases}$$

$$\begin{cases} x + y + z = 5000 \\ 2x + 4y + 5z = 21400 \\ -3y + z = -600 \end{cases}$$

Finish this by Thursday

$$\begin{cases} x + y + z = 5000 \\ 2x + 4y + 5z = 21400 \\ -3y + z = -600 \end{cases} \Rightarrow -2\begin{cases} x + y + z = 5000 \\ 2x + 4y + 5z = 21400 \end{cases}$$

NO x

$$2y + 3z = 11400$$

$$-3\begin{cases} -3y + z = -600 \\ 2y + 3z = 11400 \end{cases}$$

$$11y = 13200$$

$-3(-600) + 11400$
$= 1800 + 11400$
$= 13200$

$$y = \frac{13200}{11} \qquad \boxed{y = 1200}$$

$-3(1200) + z = -600$
$-3600 + z = -600$
$z = -600 + 3600$
$\boxed{z = 3000}$

$x + y + z = 5000$
$x + 1200 + 3000 = 5000$
$\boxed{x = 800}$

\$800 @ 2% Account,
\$1200 @ 4% " , and
\$3000 @ 5% account

I have 11 coins.

Nickels, Dimes, Quarters only.

Total value 90¢.

Dimes is three times # Quarters.

How many of each?

N → Nickels

D → Dimes

Q → Quarters

$$\begin{cases} N + D + Q = 11 \\ 5N + 10D + 25Q = 90 \quad \div 5 \\ D = 3Q \end{cases}$$

$$\begin{cases} N + D + Q = 11 \\ N + 2D + 5Q = 18 \\ D - 3Q = 0 \end{cases}$$

Finish this by Thursday

$$\begin{cases} N + D + Q = 11 \\ N + 2D + 5Q = 18 \\ D - 3Q = 0 \end{cases} \Rightarrow D = 3Q$$

$$\begin{cases} N + 3Q + Q = 11 \\ N + 2(3Q) + 5Q = 18 \end{cases}$$

$$-1 \begin{cases} N + 4Q = 11 \\ N + 11Q = 18 \end{cases}$$

$$7Q = 7$$

$$\boxed{Q = 1}$$

$$D = 3Q = 3(1) = 3$$

$$\boxed{D = 3}$$

$$N + 4Q = 11$$

$$N + 4(1) = 11$$

$$N + 4 = 11 \Rightarrow \boxed{N = 7}$$

7 Nickels, 3 Dimes, and 1 Quarters

Graph of the equation $y = ax^2 + bx + c$

contains the points $(1,10), (-1,4)$, and $(2,19)$

Find the equation.

$(1,10) \Rightarrow 10 = a(1)^2 + b(1) + c \Rightarrow a + b + c = 10$

$(-1,4) \Rightarrow 4 = a(-1)^2 + b(-1) + c \Rightarrow a - b + c = 4$

$(2,19) \Rightarrow 19 = a(2)^2 + b(2) + c \Rightarrow 4a + 2b + c = 19$

You should solve by Thursday

Find a, b, and c.

$$\begin{cases} a + b + c = 10 \\ a - b + c = 4 \\ 4a + 2b + c = 19 \end{cases}$$

$$\begin{cases} a + b + c = 10 \\ -1\ (a - b + c = 4) \end{cases}$$

$$\begin{cases} \cancel{a} + b + \cancel{c} = 10 \\ \cancel{-a} + b - \cancel{c} = -4 \end{cases}$$

$$2b = 6 \quad \boxed{b = 3}$$

$$\begin{cases} a + b + c = 10 \\ 4a + 2b + c = 19 \end{cases} \Rightarrow \begin{cases} a + 3 + c = 10 \\ 4a + 6 + c = 19 \end{cases} \Rightarrow -1 \begin{cases} a + c = 7 \\ 4a + c = 13 \end{cases}$$

$$\begin{cases} -a - \cancel{c} = -7 \\ 4a + \cancel{c} = 13 \end{cases}$$

$$3a = 6$$

$$\boxed{a = 2}$$

$$a + b + c = 10$$

$$2 + 3 + c = 10 \quad \boxed{c = 5}$$

Final Ans

$$y = 2x^2 + 3x + 5$$

$$y = ax^2 + bx + c$$

writing system of linear equations in a matrix form.

Coef. Matrix RHS

$$\begin{cases} 2x + 3y = 12 \\ x - 2y = -5 \end{cases} \Rightarrow \left[\begin{array}{cc|c} 2 & 3 & 12 \\ 1 & -2 & -5 \end{array}\right]$$

2x3

Augmented Matrix

Coef. RHS

$$\begin{cases} x + 2y - 3z = 5 \\ 3x - y \;\square = 7 \\ \square \;\; y + 4z = 0 \end{cases} \Rightarrow \left[\begin{array}{ccc|c} 1 & 2 & -3 & 5 \\ 3 & -1 & 0 & 7 \\ 0 & 1 & 4 & 0 \end{array}\right]$$

3x4

write this system in an augmented Matrix:

$$\begin{cases} 3x \;\square\; -2z = 5 \\ x - 4y \;\square = 8 \\ 2x + 3y + 4z = 0 \end{cases} \Rightarrow \left[\begin{array}{ccc|c} 3 & 0 & -2 & 5 \\ 1 & -4 & 0 & 8 \\ 2 & 3 & 4 & 0 \end{array}\right]$$

Elementary Row operations:

1) Two rows can be interchanged.

2) Multiply any row by any non Zero number.

3) Multiply any row by any non Zero number, then add to any other row.

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Consider the matrix below:

$$\left[\begin{array}{ccc|c} 3 & 18 & -21 & 12 \\ 1 & 2 & -3 & 5 \\ -2 & -3 & 4 & -6 \end{array}\right]$$

1) $R_1 \leftrightarrow R_2$

$$\left[\begin{array}{ccc|c} 1 & 2 & -3 & 5 \\ 3 & 18 & -21 & 12 \\ -2 & -3 & 4 & -6 \end{array}\right]$$

2) $\frac{1}{3}R_2 \to R_2$

$R_2 \div 3 \to R_2$

$$\left[\begin{array}{ccc|c} 1 & 2 & -3 & 5 \\ 1 & 6 & -7 & 4 \\ -2 & -3 & 4 & -6 \end{array}\right]$$

3) $2R_1 + R_3 \to R_3$

$$\left[\begin{array}{ccc|c} 1 & 2 & -3 & 5 \\ 1 & 6 & -7 & 5 \\ 0 & 1 & -2 & 4 \end{array}\right]$$

Consider the matrix below:

$$\boxed{\left[\begin{array}{ccc|c} 4 & 12 & -20 & 8 \\ 1 & 6 & -3 & 7 \\ -3 & -2 & 1 & -9 \end{array}\right]}$$

Row Equivalent

Perform the following:

1) $R_1 \leftrightarrow R_2$ then 2) $\frac{1}{4}R_2 \to R_2$, and 3) $3R_1 + R_3 \to R_3$

$R_2 \div 4 \to R_2$

$$\left[\begin{array}{ccc|c} 1 & 6 & -3 & 7 \\ 4 & 12 & -20 & 8 \\ -3 & -2 & 1 & -9 \end{array}\right] \quad \left[\begin{array}{ccc|c} 1 & 6 & -3 & 7 \\ 1 & 3 & -5 & 2 \\ -3 & -2 & 1 & -9 \end{array}\right] \quad \left[\begin{array}{ccc|c} 1 & 6 & -3 & 7 \\ 1 & 3 & -5 & 2 \\ 0 & 16 & -8 & 12 \end{array}\right]$$

Consider the matrix below:

$$\left[\begin{array}{cc|c} 4 & -3 & -15 \\ 1 & 2 & -1 \end{array}\right] \quad \left[\begin{array}{cc|c} 1 & 2 & -1 \\ 4 & -3 & -15 \end{array}\right]$$

Perform the following:

1) $R_1 \leftrightarrow R_2$

$$\left[\begin{array}{cc|c} 1 & 2 & -1 \\ 0 & -11 & -11 \end{array}\right]$$

2) $(-4)R_1 + R_2 \rightarrow R_2$

3) $R_2 \div (-11) \rightarrow R_2 \longrightarrow \left[\begin{array}{cc|c} 1 & 2 & -1 \\ 0 & 1 & 1 \end{array}\right]$

4) $(-2)R_2 + R_1 \rightarrow R_1 \longrightarrow \left[\begin{array}{cc|c} 1 & 0 & -3 \\ 0 & 1 & 1 \end{array}\right]$

Solving System of linear equations by Matrix method:

$$\begin{cases} 2x - y = -4 \\ x + 3y = 5 \end{cases}$$

① Set-up the augmented Matrix.

$$\left[\begin{array}{cc|c} 2 & -1 & -4 \\ 1 & 3 & 5 \end{array}\right]$$

Perform elementary row operations to get

$$\left[\begin{array}{cc|c} 1 & 0 & * \\ 0 & 1 & * \end{array}\right] \begin{array}{l} x = \\ y = \end{array}$$

$$\left[\begin{array}{cc|c} 2 & -1 & -4 \\ 1 & 3 & 5 \end{array}\right]$$

$R1 \leftrightarrow R2$

$$\left[\begin{array}{cc|c} 1 & 3 & 5 \\ 2 & -1 & -4 \end{array}\right]$$

$(-2)R_1 + R_2 \rightarrow R_2$

$$\left[\begin{array}{cc|c} 1 & 3 & 5 \\ 0 & -7 & -14 \end{array}\right]$$

$R_2 \div (-7) \rightarrow R_2$

$$\left[\begin{array}{cc|c} 1 & 3 & 5 \\ 0 & 1 & 2 \end{array}\right]$$

$(-3)R_2 + R_1 \rightarrow R_1$

$$\left[\begin{array}{cc|c} 1 & 0 & -1 \\ 0 & 1 & 2 \end{array}\right] \begin{array}{l} \Rightarrow x = -1 \\ \Rightarrow y = 2 \end{array}$$

Final Ans $\boxed{(-1, 2)}$ $\{(-1,2)\}$

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Solve by matrix method:

$$\begin{cases} x + y = 6 \\ x - y = 2 \end{cases} \Rightarrow \left[\begin{array}{cc|c} 1 & 1 & 6 \\ 1 & -1 & 2 \end{array}\right]$$

Pivot

$(-1)R_1 + R_2 \to R_2$

$$\left[\begin{array}{cc|c} 1 & 1 & 6 \\ 0 & -2 & -4 \end{array}\right]$$

$R_2 \div (-2) \to R_2$

$$\left[\begin{array}{cc|c} 1 & 1 & 6 \\ 0 & 1 & 2 \end{array}\right]$$

$(-1)R_2 + R_1 \to R_1$

$$\left[\begin{array}{cc|c} 1 & 0 & 4 \\ 0 & 1 & 2 \end{array}\right] \begin{array}{l} \Rightarrow x = 4 \\ \Rightarrow y = 2 \end{array}$$

Final Ans: $(4,2)$

$\{(4,2)\}$

Solve by matrix method:

$$\begin{cases} 2x + y = 3 \\ x - 3y = 12 \end{cases} \Rightarrow \left[\begin{array}{cc|c} 2 & 1 & 3 \\ 1 & -3 & 12 \end{array}\right]$$

$R_1 \leftrightarrow R_2$

$$\left[\begin{array}{cc|c} 1 & -3 & 12 \\ 2 & 1 & 3 \end{array}\right]$$

$(-2)R_1 + R_2 \to R_2$

$$\left[\begin{array}{cc|c} 1 & -3 & 12 \\ 0 & 7 & -21 \end{array}\right]$$

$R_2 \div 7 \to R_2$

$$\left[\begin{array}{cc|c} 1 & -3 & 12 \\ 0 & 1 & -3 \end{array}\right]$$

$(3)R_2 + R_1 \to R_1$

$$\left[\begin{array}{cc|c} 1 & 0 & 3 \\ 0 & 1 & -3 \end{array}\right] \begin{array}{l} x = 3 \\ y = -3 \end{array}$$

Final Ans $(3,-3)$

Solve by matrix method:

$$\begin{cases} x + y - z = -2 \\ 2x - y + z = 5 \\ -x + 2y + 2z = 1 \end{cases} \Rightarrow \left[\begin{array}{ccc|c} 1 & 1 & -1 & -2 \\ \boxed{2} & -1 & 1 & 5 \\ \boxed{-1} & 2 & 2 & 1 \end{array}\right]$$

$(-2)R_1 + R_2 \to R_2$

$R_1 + R_3 \to R_3$

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & -2 \\ 0 & -3 & 3 & 9 \\ 0 & 3 & 1 & -1 \end{array}\right]$$

$R_2 \div (-3) \to R_2$

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & -2 \\ 0 & 1 & -1 & -3 \\ 0 & 3 & 1 & -1 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & \boxed{1} & -1 & -2 \\ 0 & 1 & -1 & -3 \\ 0 & \boxed{3} & 1 & -1 \end{array}\right]$$

$(-3)R_2 + R_3 \to R_3$

$(-1)R_2 + R_1 \to R_1$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & -1 & -3 \\ 0 & 0 & 4 & 8 \end{array}\right]$$

$R_3 \div 4 \to R_3$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & -1 & -3 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

$R_3 + R_2 \to R_2$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 2 \end{array}\right] \begin{array}{l} \Rightarrow x = 1 \\ \Rightarrow y = -1 \\ \Rightarrow z = 2 \end{array}$$

Final Ans $(1, -1, 2)$

$\{(1, -1, 2)\}$

Class QZ 12

Solve for $\boxed{x \text{ only}}$ using Cramer's rule.

$$\begin{cases} 4x - 3y = 5 \\ x + y = 3 \end{cases}$$

$$D = \begin{vmatrix} 4 & -3 \\ 1 & 1 \end{vmatrix} = 4(1) - 1(-3) = \boxed{7}$$

$$D_x = \begin{vmatrix} 5 & -3 \\ 3 & 1 \end{vmatrix} = 5(1) - 3(-3) = \boxed{14}$$

$$x = \frac{D_x}{D} = \frac{14}{7} = \boxed{2} \qquad \boxed{x = 2}$$